Graph Theory Homework 4
Due: 18 June 2019 at 3:59pm as a PDF on Submitty
v1.0: Updated 12 June 2019

1. Use induction to prove that $\chi(G) \leq \Delta(G) + 1$. In other words, there will always exist a $\Delta(G) + 1$ coloring on any G.

2. Describe a construction of a 2-colorable graph G where $\forall v \in V(G) : d(v) \geq \frac{|V|-1}{2}$ for any arbitrary $|V|$.

3. When considering the two prior problems, what can you say about the dependence of maximum vertex degree on k-colorability of a general graph?

4. Use induction to prove that the chromatic polynomial of clique K_n is $\chi(K_n, k) = k(k-1)\ldots(k-n+1)$.

5. G is a connected simple graph in which every biconnected component of G is a cycle (i.e., isomorphic to cycle graph $C_n, n \geq 4$). Over all possible cut vertices $v_c \in V(G)$, the maximum number of components in $G - v_c$ is 4. Place and justify tight bounds on G's chromatic number $\chi(G)$.